

Exercise 9.3

Q1. Find the mid point of the line segment joining each of the following pairs of points.

a) $A(9,2), B(7,2)$

If $R(x, y)$ is the desired midpoint then,

$$x = \frac{x_1 + x_2}{2} = \frac{9+7}{2} = \frac{16}{2} = 8$$

$$y = \frac{y_1 + y_2}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

$$\therefore R(x, y) = R(8, 2)$$

b) $A(2,6), B(3,-6)$

If $R(x, y)$ is the desired midpoint then,

$$x = \frac{x_1 + x_2}{2} = \frac{2+3}{2} = \frac{5}{2}$$

$$y = \frac{y_1 + y_2}{2} = \frac{6-6}{2} = \frac{0}{2} = 0$$

$$R(x, y) = R\left(\frac{5}{2}, 0\right)$$

c) $A(-8,1), B(6,1)$

If $R(x, y)$ is the desired midpoint then

$$x = \frac{x_1 + x_2}{2} = \frac{-8 + 6}{2} = \frac{-2}{2} = -1$$

$$y = \frac{y_1 + y_2}{2} = \frac{1 + 1}{2} = \frac{2}{2} = 1$$

$$\therefore R(x, y) = R(-1, 1)$$

d) $A(-4, 9), B(-4, -3)$

If $R(x, y)$ is the desired mid point then,

$$x = \frac{x_1 + x_2}{2} = \frac{-4 - 4}{2} = \frac{-8}{2} = -4$$

$$y = \frac{y_1 + y_2}{2} = \frac{9 - 3}{2} = \frac{6}{2} = 3$$

$$R(x, y) = R(-4, 3)$$

e) $A(3, -11), B(3, -4)$

If $R(x, y)$ is the desired midpoint then,

$$x = \frac{x_1 + x_2}{2} = \frac{3 + 3}{2} = \frac{6}{2} = 3$$

$$y = \frac{y_1 + y_2}{2} = \frac{-11 - 4}{2} = \frac{-15}{2} = -7.5$$

$$\therefore R(x, y) = R(3, -7.5)$$

f) $A(0, 0), B(0, -5)$

If $R(x, y)$ is the desired midpoint then,

$$x = \frac{x_1 + x_2}{2} = \frac{0 + 0}{2} = 0$$

$$y = \frac{y_1 + y_2}{2} = \frac{0 - 5}{2} = \frac{-5}{2} = -2.5$$

$$\therefore R(x, y) = R(0, -2.5)$$

Q2. The end point P of a line segment PQ $(-3, 6)$ and its mid point is $(5, 8)$. Find the co-ordinates of the end point Q.

Sol: $(-3, 6)$

If $R(x, y)$ is mid point then,

$$x = \frac{x_1 + x_2}{2} \Rightarrow 5 = \frac{-3 + x_2}{2}$$

$$\Rightarrow 10 = -3 + x_2$$

$$x_2 = 10 + 3 = 13$$

$$\text{and } y = \frac{y_1 + y_2}{2} \Rightarrow 8 = \frac{6 + y_2}{2}$$

$$\Rightarrow 16 = 6 + y_2$$

$$y_2 = 10$$

$\therefore$ Coordinates of the end point $Q(13, 10)$

Q3. Prove that midpoint of the hypotenuse of a right triangle is equidistant from its three vertices

$P(-2, 5), Q(1, 3)$ and $R(-1, 0)$

$$\text{SOL. } |PQ|^2 = (1 + 2)^2 + (3 - 5)^2 = 9 + 4 = 13$$

$$|QR|^2 = (-1 - 1)^2 + (0 - 3)^2 = 4 + 9 = 13$$

$$|PR|^2 = (-1 + 2)^2 + (0 - 5)^2 = 1 + 25 = 26$$

$$\text{As } |PQ|^2 + |QR|^2 = |PR|^2$$

Hence PR is the hypotenuse

If $M(x, y)$ is desired midpoint then,

$$x = \frac{-1 + (-2)}{2} = \frac{-1 - 2}{2} = \frac{-3}{2}$$

$$y = \frac{5 + 0}{2} = \frac{5}{2}$$

$$\therefore M(x, y) = M\left(\frac{-3}{2}, \frac{5}{2}\right)$$

$$\text{Now } |PM| = \sqrt{\left(\frac{-3}{2} + 2\right)^2 + \left(\frac{5}{2} - 5\right)^2}$$

$$= \sqrt{\left(\frac{-3 + 4}{2}\right)^2 + \left(\frac{5}{2} - 10\right)^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-5}{2}\right)^2}$$

$$\begin{aligned}
 &= \sqrt{\frac{1}{4} + \frac{25}{4}} \\
 &= \sqrt{\frac{26}{4}} \\
 |RM| &= \sqrt{\left(-\frac{3}{2} + 1\right)^2 + \left(\frac{5}{2} - 0\right)^2} \\
 &= \sqrt{\left(\frac{-3+2}{2}\right)^2 + \left(\frac{5-0}{2}\right)^2} \\
 &= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{25}{4}} \\
 &= \sqrt{\frac{1+25}{4}} = \sqrt{\frac{26}{4}} \\
 |QM| &= \sqrt{\left(-\frac{3}{2} - 1\right)^2 + \left(\frac{5}{2} - 3\right)^2} \\
 &= \sqrt{\left(\frac{-3-2}{2}\right)^2 + \left(\frac{5-6}{2}\right)^2} \\
 &= \sqrt{\left(\frac{-5}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = \sqrt{\frac{25}{4} + \frac{1}{4}} = \sqrt{\frac{26}{4}}
 \end{aligned}$$

$$\text{As } |PM| = |RM| = |QM|$$

$\therefore M$ is equidistant from P , Q and R .

Q4. $O(0, 0)$, $A(3, 0)$ and $B(3, 5)$ are three points in the plane, find M_1 and M_2 as midpoints of the line segments $\overline{AB}$ and $\overline{OB}$ respectively. Find $|M_1, M_2|$.

Sol: Let $O(0,0)$, $A(3,0)$, $B(3,5)$ are three points in the plane. M_1 is the mid point of $\overline{OB}$ and M_2 is the mid-point of $\overline{AB}$

$$\begin{aligned}
 M(x, y) &= M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\
 &= M_1\left(\frac{0+3}{2}, \frac{0+5}{2}\right)
 \end{aligned}$$

$$= M_1\left(\frac{3}{2}, \frac{5}{2}\right)$$

M_2 is midpoint of $\overline{AB}$ therefore

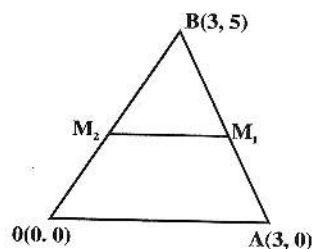
$$\begin{aligned}
 M_2\left(\frac{3+3}{2}, \frac{0+5}{2}\right) &= M_2\left(\frac{6}{2}, \frac{5}{2}\right) \\
 &= M_2\left(3, \frac{5}{2}\right)
 \end{aligned}$$

Now $\left(\frac{3}{2}, \frac{5}{2}\right)$ and $\left(3, \frac{5}{2}\right)$ are midpoints

we find $|M_1 M_2|$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned}
 \text{Then } |M_1 M_2| &= \sqrt{\left(3 - \frac{3}{2}\right)^2 + \left(\frac{5}{2} - \frac{5}{2}\right)^2} \\
 &= \sqrt{\left(\frac{6-3}{2}\right)^2 + 0} \\
 &= \sqrt{\left(\frac{3}{2}\right)^2 + 0} = \frac{3}{2}
 \end{aligned}$$



Q5. Show that the diagonals of the parallelogram having vertices

$A(1, 2)$, $B(4, 2)$, $C(-1, -3)$, $D(-4, -3)$

bisect each other.

Sol: If M_1 is desired midpoint of diagonal DB.

$$x = \frac{x_1 + x_2}{2} = \frac{4 - 4}{2} = 0$$

$$y = \frac{y_1 + y_2}{2} = \frac{2 - 3}{2} = \frac{-1}{2}$$

$$M_1(x, y) = \left(0, -\frac{1}{2}\right)$$

If M_2 is desired midpoint of diagonal AC

$$x = \frac{x_1 + x_2}{2} = \frac{1-1}{2} = 0$$

$$y = \frac{y_1 + y_2}{2} = \frac{2-3}{2} = -\frac{1}{2}$$

$$M_2(x, y) = \left(0, -\frac{1}{2}\right)$$

$\therefore$ As midpoints of the diagonals coincide
hence diagonal bisect each other.

Q6. The vertices of a triangle are $P(4,6)$, $Q(-2,-4)$ and $R(-8, 2)$ show that the length of line segment joining the mid points of line segment PR,

QR is $\frac{1}{2}$ PQ.

Sol. If M_1 is desired midpoint of line segment PR,

$$x = \frac{x_1 + x_2}{2} = \frac{4-8}{2} = -2$$

$$y = \frac{y_1 + y_2}{2} = \frac{6+2}{2} = 4$$

$$M_1(x, y) = M_1(-2, 4)$$

If M_2 is desired midpoint of line segment QR.

$$x = \frac{x_1 + x_2}{2} = \frac{-2-8}{2} = -5$$

$$y = \frac{y_1 + y_2}{2} = \frac{-4+2}{2} = -1$$

$$M_2(x, y) = M_2(-5, -1)$$

$$|M_1M_2| = \sqrt{(-5+2)^2 + (-1-4)^2}$$

$$= \sqrt{(-3)^2 + (-5)^2}$$

$$= \sqrt{9+25} = \sqrt{34}$$

$$|PQ| = \sqrt{(-2-4)^2 + (-4-6)^2}$$

$$= \sqrt{(-6)^2 + (-10)^2}$$

$$= \sqrt{36+100} = \sqrt{136} = \sqrt{34 \times 4}$$

$$= 2\sqrt{34}$$

$$\text{As } 2|M_1M_2| = |PQ|$$

$$\text{Hence } |M_1M_2| = \frac{1}{2} |PQ|$$

Review Exercise 9

Q3. Find distance between pairs of points

i) $(6,3), (3,-3)$

Let $P(6,3), Q(3,-3)$

$$|PQ| = \sqrt{(3-6)^2 + (-3-3)^2}$$

$$= \sqrt{(-3)^2 + (-6)^2}$$

$$= \sqrt{9+36} = \sqrt{45}$$

ii) $(7,5), (1,-1)$

Let $P(7,5), Q(1,-1)$

$$|PQ| = \sqrt{(7-1)^2 + (5+1)^2}$$

$$= \sqrt{(6)^2 + (6)^2} = \sqrt{36+36}$$

$$= \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$$

iii) $(0,0), (-4,-3)$

Let $P(0,0), Q(-4,-3)$

$$|PQ| = \sqrt{(-4-0)^2 + (-3-0)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

Q4. Find the midpoint between the following pairs of points.

SOL. (i) $(6,6), (4,-2)$

If $R(x, y)$ be desired midpoint, then,

$$x = \frac{6+4}{2} = \frac{10}{2} = 5$$

$$y = \frac{6-2}{2} = \frac{4}{2} = 2$$

$$R(x, y) = R(5, 2)$$

ii) $(-5, -7), (-7, -5)$

If $R(x, y)$ be desired midpoint, then,

$$x = \frac{-5-7}{2} = \frac{-12}{2} = -6$$

$$y = \frac{-5-7}{2} = \frac{-12}{2} = -6$$

$$\therefore R(x, y) = R(-6, -6)$$

iii) $(8, 0), (0, -12)$

If $R(x, y)$ be desired midpoint, then,

$$x = \frac{8+0}{2} = \frac{8}{2} = 4$$

$$y = \frac{-12+0}{2} = \frac{-12}{2} = -6$$

$$\therefore R(x, y) = R(4, -6)$$

Objective

1. Distance between points $(0, 0)$ and $(1, 1)$ is:

- (a) 0 (b) 1
(c) $\sqrt{2}$ (d) 2

2. Distance between the points $(1, 0)$ and $(0, 1)$ is:

- (a) 0 (b) 1
(c) $\sqrt{2}$ (d) 2

3. Mid-point of the points $(2, 2)$ and $(0, 0)$ is:

- (a) $(1, 1)$ (b) $(1, 0)$
(c) $(0, 1)$ (d) $(-1, -1)$

4. Mid-point of the points $(2, -2)$ and $(-2, 2)$ is:

- (a) $(2, 2)$ (b) $(-2, -2)$
(c) $(0, 0)$ (d) $(1, 1)$

5. A triangle having all sides equal is called

- (a) Isosceles (b) Scalene
(c) Equilateral (d) None of these

6. A triangle having all sides different is called:

- (a) Isosceles (b) Scalene
(c) Equilateral (d) None of these

7. The points P, Q and R are collinear if:

- (a) $|PQ| + |QR| = |PR|$
(b) $|PQ| - |QR| = |PR|$
(c) $|PQ| + |QR| = 0$
(d) None

8. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane is:

(a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, $d > 0$

(b) $d = \sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$

(c) $d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

(d) $d = \sqrt{(x_1 + x_2)^2 - (y_1 + y_2)^2}$

9. A triangle having two sides equal is called

- (a) Isosceles (b) Scalene
(c) Equilateral (d) None

10. A right triangle is that in which one of the angles has measure equal to:

(a) 80° (b) 90°

(c) 45° (d) 60°

11. In a right angle triangle ABC, Pythagoras's theorem,

(a) $|AB|^2 = |BC|^2 + |CA|^2$ where $\angle ACB = 90^\circ$.

(b) $|AB|^2 = |BC|^2 - |CA|^2$

(c) $|AB|^2 + |BC|^2 > |CA|^2$

(d) $|AB|^2 - |BC|^2 > |CA|^2$

Answer key

1.	c	2.	c	3.	a	4.	c	5.	c	6.	b
7.	a	8.	a	9.	a	10.	b	11.	a		