

# EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i)  $(x+7)(x-3) = -7$

**Solution:**  $(x+7)(x-3) = -7$

$$x^2 - 3x + 7x - 21 = -7$$

$$x^2 + 4x - 21 + 7 = 0$$

$$x^2 + 4x - 14 = 0$$

The standard form of quadratic equation is:

$$x^2 + 4x - 14 = 0$$

(ii)  $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

**Solution:**  $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

$$\frac{7(x^2 + 4) - 3x}{21} = 1$$

$$7x^2 + 28 - 3x = 21$$

$$7x^2 - 3x + 28 - 21 =$$

$$7x^2 - 3x + 7 = 0$$

The standard form of quadratic equation is:

$$7x^2 - 3x + 7 = 0$$

(iii)  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

**Solution:**  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

$$\frac{x^2 + (x+1)^2}{x(x+1)} = 6$$

$$x^2 + x^2 + 1 + 2x = 6x(x+1)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$$\Rightarrow 4x^2 + 4x - 1 = 0$$

The standard form of quadratic equation is:

$$4x^2 + 4x - 1 = 0$$

(iv)  $\left(\frac{x+4}{x-2}\right) - \left(\frac{x-2}{x}\right) + 4 = 0$

**Solution:**  $\left(\frac{x+4}{x-2}\right) - \left(\frac{x-2}{x}\right) + 4 = 0$

$$\frac{(x+4)x - (x-2)^2 + 4x(x-2)}{(x-2)(x)} = 0$$

$$x^2 + 4x - [x^2 + 2^2 - 2(x)(2)] + 4x^2 - 8x = 0$$

$$\cancel{x^2} + 4x - \cancel{x^2} - 4 + 4x + 4x^2 - 8x = 0$$

$$4x^2 + 4x + 4x - 8x - 4 = 0$$

$$4x^2 + \cancel{8x} - \cancel{8x} - 4 = 0$$

$$4x^2 - 4 = 0$$

$$4(x^2 - 1) = 0$$

$$\therefore x^2 - 1 = 0 \quad (\because 4 \neq 0)$$

So,  $x^2 - 1 = 0$  is Pure Quadratic Equation

(v)  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

**Solution:**  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

$$\frac{x(x+3) - (x+4)(x-5)}{x(x+4)} = 1$$

$$x^2 + 3x - (x^2 - 5x + 4x - 20) = 1x(x+4)$$

$$x^2 + 3x - (x^2 - 1x - 20) = x^2 + 4x$$

$$\cancel{x^2} + 3x - \cancel{x^2} + x + 20 = x^2 + 4x$$

$$3x + x + 20 = x^2 + 4x$$

$$4x + 20 = x^2 + 4x$$

$$\Rightarrow x^2 + \cancel{4x} - \cancel{4x} - 20 = 0$$

$$x^2 - 20 = 0$$

$$x^2 + 0x - 20 = 0$$

As,  $b = 0$

So,  $x^2 - 20 = 0$  is Pure Quadratic Equation

$$(vi) \quad \frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$$

$$\text{Solution: } \frac{x+1}{x+2} + \frac{x+2}{x+3} = \frac{25}{12}$$

$$\frac{(x+1)(x+3) + (x+2)(x+2)}{(x+2)(x+3)} = \frac{25}{12}$$

$$\frac{(x^2 + 3x + 1x + 3) + (x^2 + 2x + 2x + 4)}{(x+2)(x+3)} = \frac{25}{12}$$

$$\frac{(x^2 + 4x + 3) + (x^2 + 4x + 4)}{x^2 + 3x + 2x + 6} = \frac{25}{12}$$

$$\frac{2x^2 + 8x + 7}{x^2 + 5x + 6} = \frac{25}{12}$$

$$12(2x^2 + 8x + 7) = 25(x^2 + 5x + 6)$$

$$24x^2 + 96x + 84 = 25x^2 + 125x + 150$$

$$25x^2 - 24x^2 + 125x - 96x + 150 - 84 = 0$$

$$x^2 + 29x + 66 = 0$$

The standard form of quadratic equation is:

$$x^2 + 29x + 66 = 0$$

**Q.2 Solve by Factorization:**

$$(i) \quad x^2 - x - 20 = 0$$

**Solution:**

$$x^2 - x - 20 = 0$$

$$x^2 - 5x + 4x - 20 = 0$$

$$x(x-5) + 4(x-5) = 0$$

$$(x-5)(x+4) = 0$$

$$x-5=0 \quad \text{or} \quad x+4=0$$

$$x=5 \quad \text{or} \quad x=-4$$

Solution set is  $\{-4, 5\}$

$$(ii) \quad 3y^2 = y(y-5)$$

$$\text{Solution: } 3y^2 = y(y-5)$$

$$3y^2 = y^2 - 5y$$

$$3y^2 - y^2 + 5y = 0$$

$$2y^2 + 5y = 0$$

$$y(2y+5) = 0$$

$$\text{Either } y=0 \quad \text{or} \quad 2y+5=0$$

$$y=0 \quad \text{or} \quad 2y=-5$$

$$y=0 \quad \text{or} \quad y = \frac{-5}{2}$$

$$\text{Solution is } \left\{0, \frac{-5}{2}\right\}$$

$$(iii) \quad 4 - 32x = 17x^2$$

$$\text{Solution: } 4 - 32x = 17x^2$$

$$17x^2 + 32x - 4 = 0$$

$$17x^2 + 34x - 2x - 4 = 0$$

$$17x(x+2) - 2(x+2) = 0$$

$$(x+2)(17x-2) = 0$$

$$\text{Either } x+2=0 \quad \text{or} \quad 17x-2=0$$

$$x=-2 \quad \text{or} \quad 17x=2$$

$$x = -2 \quad \text{or} \quad x = \frac{2}{17}$$

$$\text{Solution set is } \left\{-2, \frac{2}{17}\right\}$$

$$(iv) \quad x^2 - 11x = 152$$

$$\text{Solution: } x^2 - 11x = 152$$

$$x^2 - 11x - 152 = 0$$

$$x^2 - 19x + 8x - 152 = 0$$

$$x(x-19) + 8(x-19) = 0$$

$$(x-19)(x+8) = 0$$

$$x-19=0 \quad \text{or} \quad x+8=0$$

$$x=19 \quad \text{or} \quad x=-8$$

$$\text{Solution set is } \{-8, 19\}$$

$$(v) \quad \frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$$

$$\text{Solution: } \frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$$

$$\frac{(x+1)^2}{x(x+1)} = \frac{25}{12}$$

$$\frac{(x)^2 + (1)^2 + 2(x)(1)}{x^2 + x} = \frac{25}{12}$$

$$\frac{x^2 + 1 + 2x + x^2}{x^2 + x} = \frac{25}{12}$$

$$12(2x^2 + 2x + 1) = 25(x^2 + x)$$

$$24x^2 + 24x + 12 = 25x^2 + 25x$$

$$0 = 25x^2 + 25x - 24x^2 - 24x - 12$$

$$0 = x^2 + x - 12$$

$$\Rightarrow x^2 + x - 12 = 0$$

$$x^2 + 4x - 3x - 12 = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x+4)(x-3) = 0$$

Either

$$x + 4 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\Rightarrow x = -4 \quad \text{or} \quad x = 3$$

$$\text{Solution set is } \{3, -4\}$$

$$(vi) \quad \frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$$

$$\text{Solution: } \frac{2}{x-9} = \frac{1}{x-3} - \frac{1}{x-4}$$

$$\frac{2}{x-9} = \frac{(x-4) - (x-3)}{(x-3)(x-4)}$$

$$\frac{2}{x-9} = \frac{\cancel{x} - 4 - \cancel{x} + 3}{x^2 - 4x - 3x + 12}$$

$$\frac{2}{x-9} = \frac{-1}{x^2 - 7x + 12}$$

$$2(x^2 - 7x + 12) = -1(x - 9)$$

$$2x^2 - 14x + 24 = -1x + 9$$

$$2x^2 - 14x + x + 24 - 9 = 0$$

$$2x^2 - 13x + 15 = 0$$

$$2x^2 - 10x - 3x + 15 = 0$$

$$2x(x-5) - 3(x-5) = 0$$

$$(x-5)(2x-3) = 0$$

$$\text{Either } x-5=0 \quad \text{or} \quad 2x-3=0$$

$$x=5 \quad \text{or} \quad 2x=3$$

$$x=5 \quad \text{or} \quad x=\frac{3}{2}$$

$$\text{Solution set is } \left\{5, \frac{3}{2}\right\}$$

**Q.3 Solve the following equations by completing square.**

$$(i) \quad 7x^2 + 2x - 1 = 0$$

$$\text{Solution: } 7x^2 + 2x - 1 = 0$$

Dividing each term of the equation by 7

$$\cancel{7}x^2 + \frac{2}{\cancel{7}}x - \frac{1}{\cancel{7}} = 0$$

$$x^2 + \frac{2}{\cancel{7}}(x) \left( \frac{\cancel{7}}{7} \right) - \frac{1}{7} = 0$$

$$(x)^2 + 2(x) \left( \frac{1}{7} \right) = \frac{1}{7}$$

Adding  $\left(\frac{1}{7}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{7+1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square root of both sides

$$\sqrt{\left(x + \frac{1}{7}\right)^2} = \pm \sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x + \frac{1}{7}\right)^2} = \pm \sqrt{\frac{4 \times 2}{49}}$$

$$x + \frac{1}{7} = \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$\therefore x = \frac{-1 \pm 2\sqrt{2}}{7}$$

$$\text{Solution set is } \left\{ \frac{-1 \pm 2\sqrt{2}}{7} \right\}$$

(ii)  $ax^2 + 4x - a = 0$  ,  $a \neq 0$

**Solution:**  $ax^2 + 4x - a = 0$  ,  $a \neq 0$

Dividing each term of the equation by 'a'

$$\frac{ax^2}{a} + 4\frac{x}{a} - \frac{a}{a} = \frac{0}{a}$$

$$x^2 + 4\frac{x}{a} - 1 = 0$$

$$x^2 + \frac{2}{\cancel{2}}(x)\left(\frac{\cancel{2}}{a}\right) = 1$$

$$(x)^2 + 2(x)\left(\frac{2}{a}\right) = 1$$

Adding  $\left(\frac{2}{a}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{2}{a}\right) + \left(\frac{2}{a}\right)^2 = 1 + \left(\frac{2}{a}\right)^2$$

$$\left(x + \frac{2}{a}\right)^2 = 1 + \frac{4}{a^2}$$

$$\left(x + \frac{2}{a}\right)^2 = \frac{a^2 + 4}{a^2}$$

Taking square root of both sides

$$\sqrt{\left(x + \frac{2}{a}\right)^2} = \pm \sqrt{\frac{a^2 + 4}{a^2}}$$

$$x + \frac{2}{a} = \pm \frac{\sqrt{a^2 + 4}}{a}$$

$$x = \frac{-2}{a} \pm \frac{\sqrt{a^2 + 4}}{a}$$

$$x = \frac{-2 \pm \sqrt{a^2 + 4}}{a}$$

$$\text{Solution set is } \left\{ \frac{-2 \pm \sqrt{a^2 + 4}}{a} \right\}$$

(iii)  $11x^2 - 34x + 3 = 0$

**Solution:**  $11x^2 - 34x + 3 = 0$

Dividing each term of the equation by 11

$$\cancel{11}x^2 - \frac{34}{11}x + \frac{3}{11} = 0$$

$$x^2 - \frac{34}{11}x = -\frac{3}{11}$$

$$(x)^2 - \frac{2}{\cancel{2}}(x)\left(\frac{\cancel{17}}{11}\right) = -\frac{3}{11}$$

$$(x)^2 - 2(x)\left(\frac{17}{11}\right) = -\frac{3}{11}$$

Adding  $\left(\frac{17}{11}\right)^2$  on both sides we get

$$(x)^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = -\frac{3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-33 + 289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{256}{121}$$

Taking square root

$$\sqrt{\left(x - \frac{17}{11}\right)^2} = \pm \sqrt{\frac{256}{121}}$$

$$x - \frac{17}{11} = \pm \frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17 \pm 16}{11}$$

$$\Rightarrow x = \frac{17-16}{11} \quad \text{or} \quad x = \frac{17+16}{11}$$

$$x = \frac{1}{11} \quad \text{or} \quad x = \frac{33}{11}$$

$$x = 3$$

Solution set is  $\left\{3, \frac{1}{11}\right\}$

(iv)  $\ell x^2 + mx + n = 0$  ,  $\ell \neq 0$

**Solution:**  $\ell x^2 + mx + n = 0$  ,  $\ell \neq 0$

Dividing each term of the equation by  $\ell$

$$\cancel{\ell} x^2 + (x)\left(\frac{m}{\cancel{\ell}}\right)x = -\frac{n}{\cancel{\ell}}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{m}{\ell}\right) = -\frac{n}{\ell}$$

$$x^2 + 2(x)\left(\frac{m}{2\ell}\right) = -\frac{n}{\ell}$$

Adding  $\left(\frac{m}{2\ell}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$\left(x + \frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x + \frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

Taking square root

$$\sqrt{\left(x + \frac{m}{2\ell}\right)^2} = \pm \sqrt{\frac{m^2 - 4\ell n}{4\ell^2}}$$

$$x + \frac{m}{2\ell} = \pm \frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = -\frac{m}{2\ell} \pm \frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = \frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}$$

Solution set is  $\left\{\frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}\right\}$

(v)  $3x^2 + 7x = 0$

**Solution:**  $3x^2 + 7x = 0$

Dividing each term of equation by 3

$$\cancel{3}x^2 + \frac{7}{\cancel{3}}x = 0$$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{3}\right) = 0$$

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) = 0$$

Adding  $\left(\frac{7}{6}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) + \left(\frac{7}{6}\right)^2 = \left(\frac{7}{6}\right)^2$$

$$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$$

Taking square root

$$\sqrt{\left(x + \frac{7}{6}\right)^2} = \pm \sqrt{\frac{49}{36}}$$

$$\left(x + \frac{7}{6}\right) = \pm \frac{7}{6}$$

$$x = \frac{-7}{6} \pm \frac{7}{6}$$

$$x = \frac{-7 \pm 7}{6}$$

$$\Rightarrow x = \frac{-7-7}{6} \quad \text{or} \quad x = \frac{-\cancel{7} + \cancel{7}}{6}$$

$$x = \frac{-14}{6} \quad \text{or} \quad x = \frac{0}{6}$$

$$x = \frac{-7}{3} \quad \text{or} \quad x = 0$$

$$\text{Solution set is } \left\{0, \frac{-7}{3}\right\}$$

$$(vi) \quad x^2 - 2x - 195 = 0$$

$$\text{Solution: } x^2 - 2x - 195 = 0$$

$$x^2 - 2x = 195$$

$$x^2 - \frac{2}{\cancel{2}}(x)\left(\frac{1}{\cancel{2}}\right) = 195$$

$$(x)^2 - 2(x)(1) = 195$$

Adding  $(1)^2$  on both sides

$$(x)^2 - 2(x)(1) + (1)^2 = 195 + (1)^2$$

$$(x-1)^2 = 195+1$$

$$(x-1)^2 = 196$$

Taking square root

$$\sqrt{(x-1)^2} = \pm \sqrt{196}$$

$$x-1 = \pm 14$$

$$x = 1 \pm 14$$

$$\Rightarrow \begin{array}{ll} x = 1-14 & \text{or} \quad x = 1+14 \\ x = -13 & \text{or} \quad x = 15 \end{array}$$

Solution set is  $\{-13, 15\}$

$$(vii) \quad -x^2 + \frac{15}{2} = \frac{7}{2}x$$

$$\text{Solution: } -x^2 + \frac{15}{2} = \frac{7}{2}x$$

$$\frac{15}{2} = x^2 + \frac{7}{2}x$$

$$\Rightarrow x^2 + \frac{7}{2}x = \frac{15}{2}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{2}\right) = \frac{15}{2}$$

$$(x)^2 + 2(x)\left(\frac{7}{4}\right) = \frac{15}{2}$$

Adding  $\left(\frac{7}{4}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{7}{4}\right) + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{15}{2} + \frac{49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{120+49}{16}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$$

Taking Square Root

$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \pm \sqrt{\frac{169}{16}}$$

$$x + \frac{7}{4} = \pm \frac{13}{4}$$

$$x = \frac{-7}{4} \pm \frac{13}{4}$$

# EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i)  $(x+7)(x-3) = -7$

Solution:  $(x+7)(x-3) = -7$

$$x^2 - 3x + 7x - 21 = -7$$

$$x^2 + 4x - 21 + 7 = 0$$

$$x^2 + 4x - 14 = 0$$

The standard form of quadratic equation is:

$$x^2 + 4x - 14 = 0$$

(ii)  $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

Solution:  $\frac{x^2 + 4}{3} - \frac{x}{7} = 1$

$$\frac{7(x^2 + 4) - 3x}{21} = 1$$

$$7x^2 + 28 - 3x = 21$$

$$7x^2 - 3x + 28 - 21 = 0$$

$$7x^2 - 3x + 7 = 0$$

The standard form of quadratic equation is:

$$7x^2 - 3x + 7 = 0$$

(iii)  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Solution:  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

$$\frac{x^2 + (x+1)^2}{x(x+1)} = 6$$

$$x^2 + x^2 + 1 + 2x = 6x(x+1)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$$\Rightarrow 4x^2 + 4x - 1 = 0$$

The standard form of quadratic equation is:

$$4x^2 + 4x - 1 = 0$$

(iv)  $\left(\frac{x+4}{x-2}\right) - \left(\frac{x-2}{x}\right) + 4 = 0$

Solution:  $\left(\frac{x+4}{x-2}\right) - \left(\frac{x-2}{x}\right) + 4 = 0$

$$\frac{(x+4)x - (x-2)^2 + 4x(x-2)}{(x-2)(x)} = 0$$

$$x^2 + 4x - [x^2 + 2^2 - 2(x)(2)] + 4x^2 - 8x = 0$$

$$\cancel{x^2} + 4x - \cancel{x^2} - 4 + 4x + 4x^2 - 8x = 0$$

$$4x^2 + 4x + 4x - 8x - 4 = 0$$

$$4x^2 + \cancel{8x} - \cancel{8x} - 4 = 0$$

$$4x^2 - 4 = 0$$

$$4(x^2 - 1) = 0$$

$$\therefore x^2 - 1 = 0 \quad (\because 4 \neq 0)$$

So,  $x^2 - 1 = 0$  is Pure Quadratic Equation

(v)  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

Solution:  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

$$\frac{x(x+3) - (x+4)(x-5)}{x(x+4)} = 1$$

$$x^2 + 3x - (x^2 - 5x + 4x - 20) = 1x(x+4)$$

$$x^2 + 3x - (x^2 - 1x - 20) = x^2 + 4x$$

$$\cancel{x^2} + 3x - \cancel{x^2} + x + 20 = x^2 + 4x$$

$$3x + x + 20 = x^2 + 4x$$

$$4x + 20 = x^2 + 4x$$

$$\Rightarrow x^2 + \cancel{4x} - \cancel{4x} - 20 = 0$$

$$x^2 - 20 = 0$$

$$x^2 + 0x - 20 = 0$$

As,  $b = 0$

So,  $x^2 - 20 = 0$  is Pure Quadratic Equation

$$(x) \quad 7(x+2a)^2 + 3a^2 = 5a(7x+23a)$$

$$\text{Solution: } 7(x+2a)^2 + 3a^2 = 5a(7x+23a)$$

$$7(x^2 + 4a^2 + 4ax) + 3a^2 = 35ax + 115a^2$$

$$7x^2 + 28a^2 + 28ax + 3a^2 - 35ax - 115a^2 = 0$$

$$7x^2 - 35ax + 28ax + 28a^2 + 3a^2 - 115a^2 = 0$$

$$7x^2 - 7ax - 84a^2 = 0$$

Dividing each term of the equation by 7 we get

$$\frac{\cancel{7}x^2}{\cancel{7}} - \frac{\cancel{7}ax}{\cancel{7}} = \frac{\cancel{12}84a^2}{\cancel{7}}$$

$$x^2 - ax = 12a^2$$

$$(x)^2 - 2(x)\left(\frac{a}{2}\right) = 12a^2$$

Adding  $\left(\frac{a}{2}\right)^2$  on both sides

$$(x)^2 - 2(x)\left(\frac{a}{2}\right) + \left(\frac{a}{2}\right)^2 = 12a^2 + \left(\frac{a}{2}\right)^2$$

$$\left(x - \frac{a}{2}\right)^2 = 12a^2 + \frac{a^2}{4}$$

$$\left(x - \frac{a}{2}\right)^2 = \frac{48a^2 + a^2}{4}$$

$$\left(x - \frac{a}{2}\right)^2 = \frac{49a^2}{4}$$

Taking square root

$$\sqrt{\left(x - \frac{a}{2}\right)^2} = \pm \sqrt{\frac{49a^2}{4}}$$

$$x - \frac{a}{2} = \pm \frac{7a}{2}$$

$$x = \frac{a}{2} \pm \frac{7a}{2}$$

$$x = \frac{a \pm 7a}{2}$$

$$\Rightarrow x = \frac{a+7a}{2} \quad \text{or} \quad x = \frac{a-7a}{2}$$

$$x = \frac{8a}{2} \quad \text{or} \quad x = \frac{-6a}{2}$$

$$x = 4a \quad \text{or} \quad x = -3a$$

Solution set is  $\{-3a, 4a\}$

## Quadratic Formula

### Derivation of Quadratic Formula By Using Completing Square Method

The quadratic equation in standard form is

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Dividing each term of the equation by 'a' we get.

$$\frac{\cancel{a}x^2}{\cancel{a}} + \frac{b}{a}x + \frac{c}{a} = 0$$

Shifting constant term  $\frac{c}{a}$  to the right, we have

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{b}{a}\right) = \frac{-c}{a}$$

$$(x)^2 + 2(x)\left(\frac{b}{2a}\right) = \frac{-c}{a}$$

Adding  $\left(\frac{b}{2a}\right)^2$  on both sides we obtain

$$(x)^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\text{or} \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking square root of the both sides, we get

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{or} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ,  $a \neq 0$  is known as "quadratic Formula"

**Example 1:** Solve the Quadratic equation

$2 + 9x = 5x^2$  by using quadratic formula.

**Solution:**  $2 + 9x = 5x^2$

The given equation in standard form can be written as

$$5x^2 - 9x - 2 = 0$$

Comparing with the standard quadratic equation  $ax^2 + bx + c = 0$ , we observe that

$$a = 5, \quad b = -9, \quad c = -2$$

Putting the values of  $a$ ,  $b$  and  $c$  in quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \text{we have}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(-2)}}{2(5)}$$

$$\text{Or } x = \frac{9 \pm \sqrt{81 + 40}}{10} = \frac{9 \pm \sqrt{121}}{10} = \frac{9 \pm 11}{10}$$

$$\text{Either } x = \frac{9+11}{10} \quad \text{or } x = \frac{9-11}{10}$$

$$x = \frac{20}{10} = 2 \quad \text{or } x = \frac{-2}{10} = -\frac{1}{5}$$

$2, -\frac{1}{5}$  are the roots of the given equation

Thus, the solution set is  $\left\{-\frac{1}{5}, 2\right\}$

**Example 2:** Using quadratic formula,

solve the equation  $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

**Solution:**  $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

Simplifying and writing in the standard form

$$\frac{(2x+1)(x+4) - (x-2)(x+2)}{(x+2)(x+4)} = 0$$

$$(2x+1)(x+4) - (x-2)(x+2) = 0$$

$$2x^2 + 8x + x + 4 - (x^2 - 4) = 0$$

$$2x^2 + 9x + 4 - x^2 + 4 = 0$$

$$\text{Or } x^2 + 9x + 8 = 0$$

$$ax^2 + bx + c = 0$$

$$\text{Here } a = 1, \quad b = 9, \quad c = 8$$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \text{we have}$$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{-9 \pm \sqrt{81 - 32}}{2} = \frac{-9 \pm \sqrt{49}}{2} = \frac{-9 \pm 7}{2}$$

$$x = \frac{-9+7}{2}$$

$$x = \frac{-2}{2} = -1$$

$$x = \frac{-9-7}{2}$$

$$x = \frac{-16}{2} = -8$$

$\therefore -1, -8$ , are the roots of the given equation. Thus, the solution set is  $\{-8, -1\}$