

EXERCISE 1.3

Q.1 $2x^4 - 11x^2 + 5 = 0$

Solution: $2x^4 - 11x^2 + 5 = 0$ (i)

Let $x^2 = y$ (ii)

$$(x^2)^2 = y^2$$

$$x^4 = y^2$$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$$2x^4 - 11x^2 + 5 = 0$$

$$2y^2 - 11y + 5 = 0$$

$$2y^2 - 10y - y + 5 = 0$$

$$2y(y - 5) - 1(y - 5) = 0$$

$$(y - 5)(2y - 1) = 0$$

$$y - 5 = 0 \quad \text{or} \quad 2y - 1 = 0$$

$$y = 0 + 5 \quad \text{or} \quad 2y = 1$$

$$y = 5 \quad \text{or} \quad y = \frac{1}{2}$$

From equation (ii) Put $y = x^2$

$$x^2 = 5 \quad \text{or} \quad x^2 = \frac{1}{2}$$

$$\sqrt{x^2} = \pm\sqrt{5} \quad \text{or} \quad \sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$$

$$x = \pm\sqrt{5} \quad \text{or} \quad x = \pm\frac{1}{\sqrt{2}}$$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Q.2 $2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$$2x^4 - 9x^2 + 4 = 0$$
 (i)

Let $x^2 = y$ (ii)

$$(x^2)^2 = y^2$$

$$x^4 = y^2$$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$$2y^2 - 9y + 4 = 0$$

$$2y^2 - 8y - 1y + 4 = 0$$

$$2y(y - 4) - 1(y - 4) = 0$$

$$(y - 4)(2y - 1) = 0$$

$$y - 4 = 0 \quad \text{or} \quad 2y - 1 = 0$$

$$y = 4 \quad \text{or} \quad 2y = 1$$

$$\Rightarrow y = 4 \quad \text{or} \quad y = \frac{1}{2}$$

From equation (ii) put $y = x^2$

$$x^2 = 4 \quad \text{or} \quad x^2 = \frac{1}{2}$$

$$\sqrt{x^2} = \pm\sqrt{4} \quad \text{or} \quad \sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$$

$$x = \pm 2 \quad \text{or} \quad x = \pm\frac{1}{\sqrt{2}}$$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$ (i)

Let $x^{\frac{1}{4}} = y$ (ii)

Taking square

$$\left(x^{\frac{1}{4}}\right)^2 = y^2$$

$$x^{\frac{1}{2}} = y^2$$

Put $x^{\frac{1}{4}} = y$ and $x^{\frac{1}{2}} = y^2$ in equation (i)

$$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$$

$$5y^2 = 7y - 2$$

$$5y^2 - 7y + 2 = 0$$

$$5y^2 - 5y - 2y + 2 = 0$$

$$5y(y - 1) - 2(y - 1) = 0$$

$$(y - 1)(5y - 2) = 0$$

$$y - 1 = 0 \quad \text{or} \quad 5y - 2 = 0$$

$$y = 1 \quad \text{or} \quad 5y = 2$$

$$y = 1 \quad \text{or} \quad y = \frac{2}{5}$$

From equation (ii) put $y = x^{\frac{1}{4}}$

$$x^{\frac{1}{4}} = 1 \quad \text{or} \quad x^{\frac{1}{4}} = \frac{2}{5}$$

$$\left(x^{\frac{1}{4}}\right)^4 = (1)^4 \quad \text{or} \quad \left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$$

$$x = 1 \quad \text{or} \quad x = \frac{2^4}{5^4}$$

$$x = \frac{16}{625}$$

Solution set is $\left\{\frac{16}{625}, 1\right\}$

$$\text{Q.4} \quad x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$

$$\text{Solution: } x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}} \quad \dots\dots\dots(i)$$

$$\text{Let } x^{\frac{1}{3}} = y \quad \dots\dots\dots(ii)$$

Taking square

$$\left(x^{\frac{1}{3}}\right)^2 = y^2$$

$$x^{\frac{2}{3}} = y^2$$

$$\text{Put } x^{\frac{1}{3}} = y \text{ and } x^{\frac{2}{3}} = y^2 \text{ in equation (i)}$$

$$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$$

$$y^2 + 54 = 15y$$

$$y^2 - 15y + 54 = 0$$

$$y^2 - 9y - 6y + 54 = 0$$

$$y(y - 9) - 6(y - 9) = 0$$

$$(y - 9)(y - 6) = 0$$

$$y - 9 = 0 \quad \text{or} \quad y - 6 = 0$$

$$y = 9 \quad \text{or} \quad y = 6$$

From equation (ii) put $y = x^{\frac{1}{3}}$

$$x^{\frac{1}{3}} = 9 \quad \text{or} \quad x^{\frac{1}{3}} = 6$$

$$\left(x^{\frac{1}{3}}\right)^3 = (9)^3 \quad \text{or} \quad \left(x^{\frac{1}{3}}\right)^3 = (6)^3$$

$$x = 9^3 \quad \text{or} \quad x = 6^3$$

$$x = 729 \quad \text{or} \quad x = 216$$

Solution set is $\{216, 729\}$

$$\text{Q.5} \quad 3x^{-2} + 5 = 8x^{-1}$$

$$\text{Solution: } 3x^{-2} + 5 = 8x^{-1} \quad \dots\dots\dots(i)$$

$$\text{Let } x^{-1} = y \quad \dots\dots\dots(ii)$$

$$(x^{-1})^2 = y^2$$

$$x^{-2} = y^2$$

$$\text{Put } x^{-1} = y \text{ and } x^{-2} = y^2 \text{ in equation (i)}$$

$$3x^{-2} + 5 = 8x^{-1}$$

$$3y^2 + 5 = 8y$$

$$3y^2 - 8y + 5 = 0$$

$$3y^2 - 5y - 3y + 5 = 0$$

$$y(3y - 5) - 1(3y - 5) = 0$$

$$(3y - 5)(y - 1) = 0$$

$$3y - 5 = 0 \quad \text{or} \quad y - 1 = 0$$

$$3y = 5 \quad \text{or} \quad y = 1$$

$$y = \frac{5}{3}$$

From equation (ii) put $y = x^{-1}$

$$x^{-1} = \frac{5}{3} \quad \text{or} \quad x^{-1} = 1$$

$$\frac{1}{x} = \frac{5}{3} \quad \text{or} \quad \frac{1}{x} = 1$$

$$x = \frac{3}{5} \quad \text{or} \quad x = 1$$

Solution set is $\left\{\frac{3}{5}, 1\right\}$

Q.6 $(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4$

Solution: $(2x^2 + 1) + \frac{3}{2x^2 + 1} = 4$ (i)

Let $2x^2 + 1 = y$ (ii)

Replacing $2x^2 + 1$ by "y" in equation (i), we get

$$y + \frac{3}{y} = 4$$

Multiplying both sides by "y"

$$y^2 + 3 = 4y$$

$$y^2 - 4y + 3 = 0$$

$$y^2 - 3y - 1y + 3 = 0$$

$$y(y - 3) - 1(y - 3) = 0$$

$$(y - 3)(y - 1) = 0$$

$$y - 3 = 0 \quad \text{or} \quad y - 1 = 0$$

$$y = 3 \quad \text{or} \quad y = 1$$

From equation (ii) Put $y = 2x^2 + 1$

$$2x^2 + 1 = 3 \quad \text{or} \quad 2x^2 + 1 = 1$$

$$2x^2 = 3 - 1 \quad \text{or} \quad 2x^2 = 1 - 1$$

$$2x^2 = 2 \quad \text{or} \quad 2x^2 = 0$$

$$x^2 = \frac{2}{2} \quad \text{or} \quad x^2 = \frac{0}{2}$$

$$x^2 = 1 \quad \text{or} \quad x^2 = 0$$

$$\sqrt{x^2} = \pm\sqrt{1} \quad \text{or} \quad \sqrt{x^2} = \pm\sqrt{0}$$

$$x = \pm 1 \quad \text{or} \quad x = 0$$

Solution set is $\{-1, 0, 1\}$

Q.7 $\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$

Solution: $\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$ (i)

Let $\frac{x}{x-3} = y$ (ii)

$$\frac{x-3}{x} = \frac{1}{y}$$

Equation (i) becomes

$$y + 4\left(\frac{1}{y}\right) = 4$$

Multiplying both sides by "y"

$$y^2 + 4 = 4y$$

$$y^2 - 4y + 4 = 0$$

$$y^2 - 2y - 2y + 4 = 0$$

$$y(y - 2) - 2(y - 2) = 0$$

$$(y - 2)(y - 2) = 0$$

$$(y - 2)^2 = 0$$

$$y - 2 = 0$$

$$y = 2$$

From equation (ii) Put $y = \frac{x}{x-3}$

$$\frac{x}{x-3} = 2$$

$$x = 2(x - 3)$$

$$x = 2x - 6$$

$$6 = 2x - x$$

$$6 = x \Rightarrow x = 6$$

Solution set is $\{6\}$

Q.8 $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$

Solution: $\frac{4x+1}{4x-1} + \frac{4x-1}{4x+1} = 2\frac{1}{6}$ (i)

Let $\frac{4x+1}{4x-1} = y$ (ii)

$$\frac{4x-1}{4x+1} = \frac{1}{y}$$

Equation (i) becomes

$$y + \frac{1}{y} = 2\frac{1}{6}$$

$$y + \frac{1}{y} = \frac{13}{6}$$

Multiplying both sides by "6y"

$$6y^2 + 6 = 13y$$

$$6y^2 - 13y + 6 = 0$$

$$6y^2 - 9y - 4y + 6 = 0$$

$$3y(2y - 3) - 2(2y - 3) = 0$$

$$(2y - 3)(3y - 2) = 0$$

$$2y - 3 = 0 \quad \text{or} \quad 3y - 2 = 0$$

$$2y = 3 \quad \text{or} \quad 3y = 2$$

$$y = \frac{3}{2} \quad \text{or} \quad y = \frac{2}{3}$$

From equation (ii) Put $y = \frac{4x+1}{4x-1}$

$$\frac{4x+1}{4x-1} = \frac{3}{2} \quad \text{or} \quad \frac{4x+1}{4x-1} = \frac{2}{3}$$

$$2(4x+1) = 3(4x-1) \quad \text{or} \quad 3(4x+1) = 2(4x-1)$$

$$8x+2 = 12x-3 \quad \text{or} \quad 12x+3 = 8x-2$$

$$2+3 = 12x-8x \quad \text{or} \quad 12x-8x = -2-3$$

$$4x = 5 \quad \text{or} \quad 4x = -5$$

$$x = \frac{5}{4} \quad \text{or} \quad x = -\frac{5}{4}$$

Solution set is $\left\{ \pm \frac{5}{4} \right\}$

Q.9 $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$

Solution: $\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$ (i)

Let $\frac{x-a}{x+a} = y$ (ii)

Or $\frac{x+a}{x-a} = \frac{1}{y}$

The equation (i) becomes

$$y - \frac{1}{y} = \frac{7}{12}$$

$$\frac{y^2 - 1}{y} = \frac{7}{12}$$

$$12(y^2 - 1) = 7y$$

$$12y^2 - 12 = 7y$$

$$12y^2 - 7y - 12 = 0$$

$$12y^2 - 16y + 9y - 12 = 0$$

$$4y(3y - 4) + 3(3y - 4) = 0$$

$$(3y - 4)(4y + 3) = 0$$

$$3y - 4 = 0 \quad \text{or} \quad 4y + 3 = 0$$

$$3y = 4 \quad \text{or} \quad 4y = -3$$

$$y = \frac{4}{3} \quad \text{or} \quad y = -\frac{3}{4}$$

From equation (ii) Putting $y = \frac{x-a}{x+a}$, we get

$$\frac{x-a}{x+a} = \frac{4}{3} \quad \text{or} \quad \frac{x-a}{x+a} = -\frac{3}{4}$$

$$3(x-a) = 4(x+a) \quad \text{or} \quad 4(x-a) = -3(x+a)$$

$$3x - 3a = 4x + 4a \quad \text{or} \quad 4x - 4a = -3x - 3a$$

$$-3a - 4a = 4x - 3x \quad \text{or} \quad 4x + 3x = -3a + 4a$$

$$-7a = x \quad \text{or} \quad 7x = a$$

$$\Rightarrow x = -7a \quad \text{or} \quad x = \frac{a}{7}$$

The solution set is $\left\{ -7a, \frac{a}{7} \right\}$

Q.10 $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Solution: $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Dividing both sides by " x^2 "

$$\frac{x^4}{x^2} - \frac{2x^3}{x^2} - \frac{2x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = \frac{0}{x^2}$$

$$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2} \right) - 2x + \frac{2}{x} - 2 = 0$$

$$\left(x^2 + \frac{1}{x^2} \right) - 2 \left(x - \frac{1}{x} \right) - 2 = 0 \quad \dots(i)$$

Let $x - \frac{1}{x} = y$ (ii)

Taking square of both sides

$$\left(x - \frac{1}{x} \right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

Putting these values in equation (i)

$$y^2 + 2 - 2(y) - 2 = 0$$

$$y^2 - 2y = 0$$

$$y(y - 2) = 0$$

$$y = 0 \quad \text{or} \quad y - 2 = 0$$

$$y = 0 \quad \text{or} \quad y = 2$$

From equation (ii) Put $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = 0 \quad \text{or} \quad x - \frac{1}{x} = 2$$

$$\frac{x^2 - 1}{x} = 0 \quad \text{or} \quad \frac{x^2 - 1}{x} = 2$$

$$x^2 - 1 = 0 \quad \text{or} \quad x^2 - 1 = 2x$$

$$x^2 = 1 \quad \text{or} \quad x^2 - 2x - 1 = 0$$

$$\sqrt{x^2} = \pm\sqrt{1} \quad \text{or} \quad \boxed{1x^2 - 2x - 1 = 0}$$

$$\boxed{x = \pm 1}$$

Now Solving $1x^2 - 2x - 1 = 0$ by quadratic formula

$$a = 1, \quad b = -2, \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2(1 \pm \sqrt{2})}{2}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

The solution set is $\{\pm 1, 1 \pm \sqrt{2}\}$

$$\text{Q.11} \quad 2x^4 + x^3 - 6x^2 + x + 2 = 0$$

$$\text{Solution:} \quad 2x^4 + x^3 - 6x^2 + x + 2 = 0$$

Dividing each terms by x^2

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \dots\dots(i)$$

$$\text{Let} \quad x + \frac{1}{x} = y \quad \dots\dots(ii)$$

Taking square of both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2(\cancel{x})(\cancel{\frac{1}{x}}) = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Put $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in equation (i)

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$$

$$2(y^2 - 2) + y - 6 = 0$$

$$2y^2 - 4 + y - 6 = 0$$

$$2y^2 + y - 10 = 0$$

$$2y^2 + 5y - 4y - 10 = 0$$

$$y(2y + 5) - 2(2y + 5) = 0$$

$$(2y + 5)(y - 2) = 0$$

$$2y + 5 = 0 \quad \text{or} \quad y - 2 = 0$$

$$2y = -5 \quad \text{or} \quad y = 2$$

$$y = \frac{-5}{2}$$

From equation (ii) Put $y = x + \frac{1}{x}$

$$x + \frac{1}{x} = \frac{-5}{2}$$

$$\frac{x^2 + 1}{x} = \frac{-5}{2}$$

$$2(x^2 + 1) = -5x$$

$$2x^2 + 2 = -5x$$

$$2x^2 + 5x + 2 = 0$$

$$2x^2 + 4x + 1x + 2 = 0$$

$$2x(x + 2) + 1(x + 2) = 0$$

$$(x + 2)(2x + 1) = 0$$

$$x + 2 = 0 \text{ or } 2x + 1 = 0$$

$$x = -2 \text{ or } 2x = -1$$

$$\boxed{x = -2} \text{ or } \boxed{x = \frac{-1}{2}}$$

$$x + \frac{1}{x} = 2$$

$$\frac{x^2 + 1}{x} = 2$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$x^2 - 1x - 1x + 1 = 0$$

$$x(x - 1) - 1(x - 1) = 0$$

$$(x - 1)(x - 1) = 0$$

$$(x - 1)^2 = 0$$

$$\sqrt{(x - 1)^2} = \sqrt{0}$$

$$x - 1 = 0$$

$$x = 1$$

Solution set is $\left\{1, -2, \frac{-1}{2}\right\}$

Q.12 $4.2^{2x+1} - 9.2^x + 1 = 0$

Solution: $4.2^{2x+1} - 9.2^x + 1 = 0$

$$4.2^{2x} 2^1 - 9.2^x + 1 = 0$$

$$8(2^x)^2 - 9(2^x) + 1 = 0 \dots\dots\dots (i)$$

Let $2^x = y \dots\dots\dots (ii)$

The equation (i) becomes

$$8y^2 - 9y + 1 = 0$$

$$8y^2 - 8y - 1y + 1 = 0$$

$$8y(y - 1) - 1(y - 1) = 0$$

$$(y - 1)(8y - 1) = 0$$

$$y - 1 = 0 \quad \text{or} \quad 8y - 1 = 0$$

$$y = 1 \quad \text{or} \quad 8y = 1$$

$$\text{or} \quad y = \frac{1}{8}$$

From equation (ii) Put $y = 2^x$

$$2^x = 1 \quad \text{or} \quad 2^x = \frac{1}{8}$$

$$2^x = 2^0 \quad \text{or} \quad 2^x = \frac{1}{2^3}$$

$$\Rightarrow x = 0 \quad \text{or} \quad 2^x = 2^{-3} \Rightarrow x = -3$$

Solution set is $\{-3, 0\}$

Q.13 $3^{2x+2} = 12.3^x - 3$

Solution: $3^{2x+2} = 12.3^x - 3$

$$3^{2x} 3^2 - 12.3^x + 3 = 0$$

$$9(3^x)^2 - 12.(3^x) + 3 = 0 \dots\dots\dots (i)$$

Let $3^x = y \dots\dots\dots (ii)$

Put $3^x = y$ in equation (i)

$$9y^2 - 12y + 3 = 0$$

$$9y^2 - 9y - 3y + 3 = 0$$

$$9y(y - 1) - 3(y - 1) = 0$$

$$(y - 1)(9y - 3) = 0$$

$$y - 1 = 0 \quad \text{or} \quad 9y - 3 = 0$$

$$y = 1 \quad \text{or} \quad 9y = 3$$

$$\boxed{y = 1} \quad \text{or} \quad y = \frac{3}{9} \Rightarrow \boxed{y = \frac{1}{3}}$$

From equation (ii) Put $y = 3^x$

$$3^x = 1 \quad \text{or} \quad 3^x = \frac{1}{3}$$

$$3^x = 3^0 \quad \text{or} \quad 3^x = 3^{-1}$$

$$\Rightarrow x = 0 \quad \text{or} \quad x = -1$$

The solution set is $\{0, -1\}$

Q.14 $2^x + 64.2^{-x} - 20 = 0$

Solution: $2^x + 64.2^{-x} - 20 = 0$

$$2^x + \frac{64}{2^x} - 20 = 0 \dots\dots\dots (i)$$

Let $2^x = y \dots\dots\dots (ii)$

Put $2^x = y$ in equation (i)

$$y + \frac{64}{y} - 20 = 0$$

Multiply both sides by "y"

$$y^2 + 64 - 20y = 0$$

$$y^2 - 20y + 64 = 0$$

$$y^2 - 16y - 4y + 64 = 0$$

$$y(y - 16) - 4(y - 16) = 0$$

$$(y - 16)(y - 4) = 0$$

$$y - 16 = 0 \quad \text{or} \quad y - 4 = 0$$

$$y = 16 \quad \text{or} \quad y = 4$$

From equation (ii) put $y = 2^x$

$$2^x = 16 \quad \text{or} \quad 2^x = 4$$

$$2^x = 2^4 \quad \text{or} \quad 2^x = 2^2$$

$$\Rightarrow x = 4 \quad \text{or} \quad x = 2$$

The solution set is $\{2, 4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7) = 192$

Solution: $(x+1)(x+3)(x-5)(x-7) = 192$

$$\begin{array}{l} \therefore a+b=c+d \\ 1-5=3-7 \\ -4=-4 \end{array}$$

$$(x+1)(x-5)(x+3)(x-7) = 192$$

$$(x^2 - 5x + 1x - 5)(x^2 - 7x + 3x - 21) = 192$$

$$(x^2 - 4x - 5)(x^2 - 4x - 21) = 192 \dots(i)$$

Let $x^2 - 4x = y$ (ii)

The equation (i) becomes

$$(y-5)(y-21) = 192$$

$$y^2 - 21y - 5y + 105 = 192$$

$$y^2 - 26y + 105 - 192 = 0$$

$$y^2 - 26y - 87 = 0$$

$$y^2 - 29y + 3y - 87 = 0$$

$$y(y-29) + 3(y-29) = 0$$

$$(y-29)(y+3) = 0$$

$$y-29=0 \quad \text{or} \quad y+3=0$$

$$y=29 \quad \text{or} \quad y=-3$$

From equation (ii) Put $y = x^2 - 4x$

$$x^2 - 4x = 29 \quad \text{or} \quad x^2 - 4x = -3$$

$$x^2 - 4x - 29 = 0 \quad \text{or} \quad x^2 - 4x + 3 = 0$$

$$\boxed{x^2 - 4x - 29 = 0} \quad \text{or} \quad \boxed{x^2 - 4x + 3 = 0}$$

First we solve $x^2 - 4x - 29 = 0$ by quadratic formula

$$a=1, \quad b=-4, \quad c=-29$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-29)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 + 116}}{2}$$

$$x = \frac{4 \pm \sqrt{132}}{2}$$

$$x = \frac{4 \pm \sqrt{4 \times 33}}{2}$$

$$x = \frac{4 \pm 2\sqrt{33}}{2}$$

$$x = \frac{\cancel{2} (2 \pm \sqrt{33})}{\cancel{2}}$$

$$\boxed{x = 2 \pm \sqrt{33}}$$

Now we solve $x^2 - 4x + 3 = 0$ by factorization

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3) - 1(x-3) = 0$$

$$(x-3)(x-1) = 0$$

$$x-3=0 \quad \text{or} \quad x-1=0$$

$$\boxed{x=3} \quad \text{or} \quad \boxed{x=1}$$

Solution set is $\{1, 3, 2 \pm \sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

Solution: $(x-1)(x-2)(x-8)(x+5) + 360 = 0$

$$[(x-1)(x-2)][(x-8)(x+5)] + 360 = 0$$

$$[x^2 - 2x - 1x + 2][x^2 + 5x - 8x - 40] + 360 = 0$$

$$(x^2 - 3x + 2)(x^2 - 3x - 40) + 360 = 0 \dots(i)$$

Let $x^2 - 3x = y$ (ii)

Put it in equation (i)

$$(y+2)(y-40) + 360 = 0$$

$$y^2 - 40y + 2y - 80 + 360 = 0$$

$$y^2 - 38y + 280 = 0$$

$$y^2 - 28y - 10y + 280 = 0$$

$$y(y-28) - 10(y-28) = 0$$

$$(y-28)(y-10) = 0$$

$$y-28=0 \quad \text{or} \quad y-10=0$$

$$y=28 \quad \text{or} \quad y=10$$

From equation (ii) Put $y = x^2 - 3x$

$$x^2 - 3x = 28$$

$$x^2 - 3x - 28 = 0$$

$$x^2 - 7x + 4x - 28 = 0$$

$$x(x-7) + 4(x-7) = 0$$

$$(x-7)(x+4) = 0$$

$$x-7=0 \quad \text{or} \quad x+4=0$$

$$x=7 \quad \text{or} \quad x=-4$$

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0$$

$$x(x-5) + 2(x-5) = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0 \quad \text{or} \quad x+2=0$$

$$x=5 \quad \text{or} \quad x=-2$$

The solution set is

$$\{5, -2, 7, -4\}$$

Radical Equation

An equation involving expression of the variable under radical sign is called radical equation. For example $\sqrt{x+1} = 2$

(i) The equation of the type $\sqrt{ax+b} = cx+d$

Example 1:

Solve the equation $\sqrt{3x+7} = 2x+3$

Solution: $\sqrt{3x+7} = 2x+3$ (i)

Squaring both sides of the equation (i), we get

$$(\sqrt{3x+7})^2 = (2x+3)^2$$

$$3x+7 = (2x)^2 + 2(2x)(3) + (3)^2$$

$$3x+7 = 4x^2 + 12x + 9$$

$$0 = 4x^2 + 12x + 9 - 3x - 7$$

$$\Rightarrow 4x^2 + 9x + 2 = 0$$

Applying quadratic formula,

$$a = 4, b = 9, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 4 \times 2}}{2 \times 4}$$

$$x = \frac{-9 \pm \sqrt{81 - 32}}{8} = \frac{-9 \pm \sqrt{49}}{8} = \frac{-9 \pm 7}{8}$$

$$x = \frac{-9+7}{8} \quad \text{or} \quad x = \frac{-9-7}{8}$$

$$x = \frac{-2}{8} \quad \text{or} \quad x = \frac{-16}{8}$$

$$x = \frac{-1}{4} \quad \text{or} \quad x = -2$$

Checking: Putting $x = -\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right) + 7} = 2\left(-\frac{1}{4}\right) + 3$$

$$\Rightarrow \sqrt{\frac{-3+28}{4}} = -\frac{1}{2} + 3$$

$$\Rightarrow \sqrt{\frac{25}{4}} = \frac{-1+6}{2}$$

$$\frac{5}{2} = \frac{5}{2} \quad \text{which is true.}$$

Putting $x = -2$ in the equation (i), we get

$$\sqrt{3(-2) + 7} = 2(-2) + 3$$

$$\Rightarrow \sqrt{1} = -1$$

$$1 \neq -1$$

On checking, we find that $x = -2$ does not satisfy the equation (i), so it is an extraneous root. Thus the solution set is $\left\{-\frac{1}{4}\right\}$

(ii) The equation of the type $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$

Example 2: Solve the equation

$$\sqrt{x+3} + \sqrt{x+6} = \sqrt{x+11}$$

Solution: $\sqrt{x+3} + \sqrt{x+6} = \sqrt{x+11}$ (i)

Squaring both sides of the equation (i), we have

$$(\sqrt{x+3} + \sqrt{x+6})^2 = (\sqrt{x+11})^2$$

$$(\sqrt{x+3})^2 + (\sqrt{x+6})^2 + 2(\sqrt{x+3})(\sqrt{x+6}) = x+11$$

$$x+3 + x+6 + 2(\sqrt{x+3})(\sqrt{x+6}) = x+11$$

$$2x+9 + 2\sqrt{(x+3)(x+6)} = x+11$$

$$2\sqrt{x^2+6x+3x+18} = x+11-2x-9$$

$$\text{or} \quad 2\sqrt{x^2+9x+18} = -x+2$$

Again taking square of both sides we get

$$(2)^2 (\sqrt{x^2+9x+18})^2 = (-x)^2 + (2)^2 + 2(-x)(2)$$

$$4(x^2+9x+18) = x^2+4-4x$$

$$(4x^2+36x+72) = x^2-4x+4$$

$$4x^2+36x+72-x^2+4x-4=0$$

$$3x^2+40x+68=0$$

Applying quadratic formula

$$a = 3, b = 40, c = 68$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-40 \pm \sqrt{(40)^2 - 4(3)(68)}}{2(3)}$$

$$x = \frac{-40 \pm \sqrt{1600 - 816}}{6}$$

$$x = \frac{-40 \pm \sqrt{784}}{6} = \frac{-40 \pm 28}{6}$$

We have

$$x = \frac{-40 + 28}{6}$$

$$x = \frac{-12}{6}$$

$$x = -2$$

$$x = \frac{-40 - 28}{6}$$

$$x = \frac{-68}{6}$$

$$x = \frac{-34}{3}$$

Checking:

Putting $x = -2$ in (i)

$$\sqrt{-2+3} + \sqrt{-2+6} = \sqrt{-2+11}$$

$$\sqrt{1} + \sqrt{4} = \sqrt{9}$$

$$1 + 2 = 3$$

$$3 = 3 \quad \text{which is true}$$

So one root is 3

Now Putting $x = \frac{-34}{3}$ in the equation (i), we have

$$\sqrt{\frac{-34}{3} + 3} + \sqrt{\frac{-34}{3} + 6} = \sqrt{\frac{-34}{3} + 11}$$

or

$$\sqrt{\frac{-34+9}{3}} + \sqrt{\frac{-34+18}{3}} = \sqrt{\frac{-34+33}{3}}$$

$$\Rightarrow \sqrt{\frac{25}{3} \times (-1)} + \sqrt{\frac{16}{3} \times (-1)} = \sqrt{\frac{1}{3} \times (-1)}$$

$$\Rightarrow \frac{5}{\sqrt{3}}i + \frac{4}{\sqrt{3}}i = \frac{1}{\sqrt{3}}i \quad (\because i = \sqrt{-1})$$

$$\frac{9}{\sqrt{3}}i = \frac{1}{\sqrt{3}}i \quad \text{which is not true.}$$

As $\frac{-34}{3}$ is an extraneous root,

So, the solution set is $\{-2\}$.

(iii) The equation of the type $\sqrt{x^2+px+m} + \sqrt{x^2+px+n} = q$

Example 3: Solve the equation

$$\sqrt{x^2-3x+36} - \sqrt{x^2-3x+9} = 3$$

Solution: $\sqrt{x^2-3x+36} - \sqrt{x^2-3x+9} = 3 \dots (i)$

Let $x^2-3x = y \dots \dots \dots (ii)$

Put it in equation (i)

$$\text{Then } \sqrt{y+36} - \sqrt{y+9} = 3$$

Squaring both sides

$$(\sqrt{y+36} - \sqrt{y+9})^2 = (3)^2$$

$$(\sqrt{y+36})^2 + (\sqrt{y+9})^2 - 2(\sqrt{y+36})(\sqrt{y+9}) = 9$$

$$y+36 + y+9 - 2\sqrt{(y+36)(y+9)} = 9$$

$$2y+45 - 2\sqrt{(y^2+36y+9y+324)} = 9$$

$$-2\sqrt{(y^2+45y+324)} = 9-2y-45$$

$$-2\sqrt{y^2+45y+324} = -2y-36$$

$$\cancel{-2}\sqrt{y^2+45y+324} = \cancel{-2}(y+18)$$

Again squaring both sides

$$(\sqrt{y^2+45y+324})^2 = (y+18)^2$$

$$y^2+45y+324 = (y)^2 + 2(y)(18) + (18)^2$$

$$\cancel{y^2} + 45y + \cancel{324} = \cancel{y^2} + 36y + \cancel{324}$$

$$45y - 36y = 0$$

$$9y = 0$$

$$\Rightarrow y = 0$$

From equation (ii) put $y = x^2-3x$ so

$$x^2-3x = 0$$

$$\Rightarrow x(x-3) = 0$$

Either $x = 0$ or $x-3 = 0 \Rightarrow x = 3$

$\therefore x = 0, 3$ are the roots of the equation.

Thus, the solution set is $\{0, 3\}$